Data 102, Spring 2025
Midterm 2

- You have **110 minutes** to complete this exam. There are **6 questions**, totaling **50 points**.
- You may use **one** 8.5×11 sheet of handwritten notes (front and back), and the provided reference sheet. No other notes or resources are allowed.
- You should write your solutions inside this exam sheet.
- You should write your Student ID on every sheet (in the provided blanks).
- Make sure to write clearly. We can't give you credit if we can't read your solutions.
- Even if you are unsure about your answer, it is better to write down something so we can give you partial credit.
- We have provided a blank page of scratch paper **at the end** of the exam. No work on this page will be graded.
- You may, without proof, use theorems and facts given in the discussions or lectures, **but please cite them**.
- We don't answer questions individually. If you believe something is unclear, bring your question to us and if we find your question valid we will make a note to the whole class.
- Unless otherwise stated, no work or explanations will be graded for multiple-choice questions.
- Unless otherwise stated, you must show your work for free-response questions in order to receive credit.

Last name	
First name	
Student ID (SID) number	
Berkeley email	
Name of person to your left	
Name of person to your right	

Honor Code [1 pt]:
As a member of the UC Berkeley community, I act with honesty, integrity, and respect for others. I am the person whose name is on the exam, and I completed this exam in accordance with the Honor Code.

Signature: ______________________________

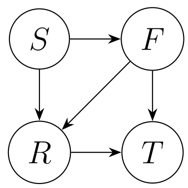
1 True or False and Multiple Choice [6 Pts]

For (a) - (d), determine whether the statement is true or false. For all parts of this question, no work will be graded and no partial credit will be assigned.

(a) [1 Pt] Backpropagation is an algorithm for efficiently computing gradients of functions.

● **True** ◯ False

(b) [1 Pt] In the causal graph below, T is a collider for estimating the treatment effect of S on R.



● **True** ◯ False

(c) [1 Pt] In multi-armed bandits, the regret incurred when using the explore-then-commit (ETC) algorithm grows logarithmically as a function of the number of rounds.

● **True** ◯ False

(d) [1 Pt] If we are able to guarantee that we collect data on every single confounding variable in a causal question, then we can always use matching to produce an unbiased estimate of the average treatment effect (ATE).

◯ True ● **False**

(e) [2 Pts] For each statement about intervals, determine whether it applies to confidence intervals, credible intervals, both, or neither.

(i) Suppose a 95% ________ interval for a coefficient is $[0.7, 1.5]$. Changing 95% to 90% will make the interval wider (i.e., increase the difference between the upper and lower bound).

◯ Confidence ◯ Credible ◯ Both ● **Neither**

(ii) In a GLM, the ________ interval for a coefficient depends on the prior distribution for that coefficient.

◯ Confidence ● **Credible** ◯ Both ◯ Neither

2 Reinforcement Learning [5 Pts]

Consider a Markov Decision Process (MDP) with five states and the actions "left" and "right". States 1 and 5 are terminal states. At states 2-4, taking an action deterministically moves in that direction. The reward for reaching state 1 is 10, and the reward for reaching state 5 is 100.

1	2	3	4	5

(a) [2 Pts] For this part only, suppose the robot's programming is faulty, and when taking the left action, the robot is equally likely to stay in place, move right, or move left. Which of the following part(s) of the MDP must change to incorporate this information? Select all answers that apply **by filling in the square next to each correct answer**.

- ☐ Set of states
- ☐ Set of actions
- ■ **Transition probabilities**
- ☐ Rewards
- ☐ Start and/or terminal state(s)
- ☐ None of the above

(b) [2 Pts] Suppose the discount factor is 0.001. What is the optimal policy for state 2? Explain your answer in one sentence or less: you should need very little calculation to answer this question.

Optimal policy for state 2: ________________________

Explanation:

Solution:

Optimal policy: left

Explanation: with such a high discount factor, any reward we obtain from state 5 will be discounted by at least 0.001^2, which will be much less than the immediate reward we get from moving to state 1. So, we should move left to maximize our reward.

(c) [1 Pt] Suppose the discount factor is 0.99. What is the optimal policy for state 2? Explain your answer in one sentence or less: you should need very little calculation to answer this question.

Optimal policy for state 2: ________________________

Explanation:

Solution: Optimal policy: right

Moving to the right three times will obtain a reward of $(0.99)^2 \times 100$, which is greater than the reward obtained from moving left (10). So, we should move right.

3 Vending Machine Models [11 Pts]

Sandya is looking at vending machines on campus. She uses the number of unique snacks in a machine (x_1, measured when the machine is restocked in the morning) and the average cost of a snack (x_2) to predict the number of items sold from that machine (y) in a day.

(a) [2 Pts] Sandya defines a GLM with average prediction $\hat{\mu}(x_1, x_2) = \exp(\beta_1 x_1 + \beta_2 x_2)$ and likelihood model $y \sim \text{Poisson}(\hat{\mu})$. She sets $\beta_1 = 7$ and $\beta_2 = -2$. Which of the following statements are accurate statements about Sandya's model? Select all answers that apply **by filling in the square next to each correct answer**. You may use the fact that $\exp(-2) \approx 0.14$.

- ☐ For every increase in the average price by $\$1$, the model predicts that (on average) the machine will sell two fewer items.
- ■ **For every increase in the average price by $\$1$, the model predicts that (on average) the machine will sell 14% as many items.**
- ■ **If the variance of y is much larger than the mean of y, then this model will be overdispersed.**

> **Solution:** Increasing x_1 by 1 gives $\mu' = 5^{\beta_1(x_1+1)+\beta_2 x_2} = 5^{\beta_1} * 5^{\beta_1 x_1 + \beta_2 x_2} = 5^{\beta_1} * \mu$
> Thus every new drink added to the vending machine causes the average rating to increase by a multiplicative factor of 5^{β_1}

For the remainder of this question, Sandya experiments with two more predictors x_3 and x_4, and fits various **Poisson GLMs** from data using different combinations of predictors x_1 through x_4. She identifies three models that give her good accuracy, and makes the following table summarizing their performance:

Model	β_1	β_2	β_3	β_4	log-likelihood
A	6.0	-1.3	not used	not used	-1043.0
B	4.1	-0.2	1.8	5	-1042.0
C	5.0	-1.0	2.6	not used	-1041.5

(b) [2 Pts] Sandya wants to use the Akaike Information Criterion (AIC) to choose the model that will generalize best to new data. Based on this, which model should she choose and why? **You must show your work to receive credit.**

> **Solution:** The AIC is equal to $-2(\text{Log-Likelihood}) + 2(\text{ \# of coefficients})$. The three models have AICs of:
>
> $$\begin{aligned} AIC_A &= -2(-1043) + 4 && = 2090 \\ AIC_B &= -2(-1042) + 8 && = 2092 \\ AIC_C &= -2(-1041.5) + 4 && = 2087 \end{aligned}$$

Smaller values of AIC indicate better generalization, so we choose the model with the smallest AIC: Model C.

(c) [2 Pts] For a particular vending machine, suppose $x_1 = 15$, $x_2 = 2.9$, $x_3 = 6$, and $x_4 = 0.7$. Write an expression for the average prediction that Model C would make for this vending machine. Your expression should contain only numbers and mathematical expressions (no variables), but you do not need to simplify.

Solution: $\bar{y} = \exp\left(5 \times 15 + (-1) \times 2.9 + 2.6 \times 6\right)$

(d) [2 Pts] Sandya carries out a posterior predictive check (PPC) for Model A. Which of the following must be true? Select all answers that apply **by filling in the square next to each correct answer**.

- ☐ Part of a PPC involves comparing posterior samples for the coefficients β_1 and β_2 to see if they are reasonable.
- ■ **If the posterior predictive samples have a very similar distribution to the observed number of items sold, then Sandya should conclude that the model is a good fit for her training set.**
- ☐ If the posterior predictive samples have a very similar distribution to the observed number of items sold, then Sandya should conclude that the model will generalize well to vending machines from other college campuses.

(e) [3 Pts] Sandya trains two more models on a larger set of features x_1 through x_30, and compares them to each other. Model G is a Poisson GLM, and Model R is a random forest trained on the same data using the same four features. Which of the following statements, if true, will make Model G a better choice than Model R? Select all answers that apply **by filling in the square next to each correct answer**.

- ☐ Sandya's top priority is choosing the model that will give the most accurate predictions.
- ☐ Sandya wants to balance accuracy and interpretability, and on the training set, Model G has very high error and Model R has very low error.
- ■ **Sandya wants to balance accuracy and interpretability, and on the test set, both Model G and Model R have very low error.**

4 Colorful Exams (11 pts)

Kobe and Ruhi are two GSIs in the same class who want to know whether using blue paper (instead of white) causes students to score better on exams. Each GSI has 20 students in their section, and there are no other GSIs or sections in the class.

Kobe has eleven exams printed on blue paper, and Ruhi has eight. Within each section, the GSI **randomly assigns** students to either take their final exam on blue paper (treatment, $Z_i = 1$) or on white paper (control, $Z_i = 0$). They define Y_i as the student's final exam score. They gather the following data:

GSI	Number of treated students	Treatment mean score	Control mean score
Kobe	11	75	60
Ruhi	8	90	80

(a) [2 Pts] Explain why a student's section/GSI is a confounding variable, in two sentences or less. Your answer must use only the information provided: additional assumptions or speculation will not receive credit for this question.

Solution: A student's section affects the treatment, their chance of receiving treatment (students in Ruhi's section are less likely to get blue paper); and also affects the outcome (students in Ruhi's section achieve higher scores regardless of treatment). So, section/GSI is a confounding variable.

(b) [3 Pts] Which of the following must be true, based on the information provided? Select all answers that apply **by filling in the square next to each correct answer**.

■ **Conditioned on a student being in Kobe's section, receiving treatment is independent of the pair of potential outcomes.**

■ **The potential outcome $Y_i(1)$ represents the score that student i would receive if they took the final exam on blue paper.**

☐ Ruhi's teaching causes her students to do better on the final exam than Kobe's students.

(c) [1 Pt] Let X_i denote each student's section ($X_i = K$ for students in Kobe's section and

$X_i = R$ otherwise). Compute the propensity scores $e(K)$ and $e(R)$.

$e(K) =$

$e(R) =$

Solution: $e(K) = \frac{11}{20}$ and $e(R) = 8/20 = 2/5$.

(d) [2 Pts] Compute the inverse propensity weighted (IPW) estimate for the causal effect of paper color on final exam score. You may leave your answer in terms of the propensity scores $e(K)$ and $e(R)$, but there should be no other variables (only numbers) in your answer. You do not need to simplify.

Solution:

$$\begin{aligned}\hat{\tau}_{IPW} &= \frac{1}{n}\left[\sum_{Z_i=1}\frac{Y_i}{e(X_i)} - \sum_{Z_i=0}\frac{Y_i}{1-e(X_i)}\right] \\ &= \frac{1}{40}\left[10\frac{75}{e(K)} + 8\frac{90}{e(R)} - 10\frac{60}{1-e(K)} - 12\frac{80}{1-e(R)}\right]\end{aligned}$$

(e) [1 Pt] Kobe and Ruhi correctly compute the answer from part (d). Is their result an unbiased estimate of the average treatment effect (ATE) of using blue paper on final exam score? Explain why or why not.

- ● **Yes**
- ○ No

Explanation:

Solution: Conditioned on a student's section assignment, the treatment decisions are randomized, so the unconfoundedness assumption is satisfied. So, using IPW gives us an unbiased estimate of the ATE.

Note: the last part of this question is unrelated to any of the previous parts, and asks about instrumental variables in general.

(f) [2 Pts] In general, which of the following conditions are **necessary** to use an instrumental variable W with treatment Z, outcome Y, and (unobserved) confounder X? Select all answers that apply **by filling in the square next to each correct answer**.

■ $Y \perp\!\!\!\perp W \mid Z$

□ $\text{Cov}[W, Z] > 0$

■ $\text{Cov}[Z, X] = 0$

□ $W \perp\!\!\!\perp X \mid Z$

Solution:

- Exclusion Restriction
- We only need $\text{Cov}[W, Z] \neq 0$
- We need $Z \perp\!\!\!\perp X \implies \text{Cov}[Z, X] = 0$
- $W \perp\!\!\!\perp X, W \not\perp\!\!\!\perp X \mid Z$ because Z is a collider for W and Z

5 Ice Cream Conundrum [10 Pts]

Clara opens a weekly ice cream pop-up in Berkeley. Every Saturday, she makes 100 scoops of ice cream and sells as many as she can. The more scoops she sells, the more money she makes. Each week, she makes **exactly one flavor** from the following list: vanilla, mango, strawberry, coconut.

Her friend suggests using multi-armed bandits to decide on the best flavor to make each week. They define the weekly reward as the number of scoops she sells that week.

(a) [2 Pts] Describe one assumption she is making by using multi-armed bandits that might not be true, and explain why. Your answer must be two sentences or less.

Solution: Multi-armed bandits assume stationary rewards, but people's preferences may change over time (based on trends, people's moods, hype about certain flavors, etc.).

For the remainder of this question, assume that any conditions necessary to use multi-armed bandits are satisfied.

(b) [2 Pts] For this part only, Clara conducts a survey of 15 representative likely customers on which of her four flavors they are most likely to buy. Clara is concerned about the small sample size, but still wants to use the survey results in her decisions. Given this, which algorithm is the best choice for Clara to use? Choose the single best answer **by filling in the circle next to it.** You must explain your answer in two sentences or less to receive full credit.

- ○ Explore-then-Commit (ETC)
- ○ Upper Confidence Bound (UCB)
- ● **Thomson Sampling (TS)**

Explanation:

Solution: Thomson sampling lets us use the information from the survey as a **prior**, which neither of the other two methods can do.

No work or answers below this line will be graded.

For the remainder of the question, assume that Clara uses one of the three bandit algorithms you learned about (ETC, UCB, TS). After 26 weeks, she summarizes her sales in the following table:

Flavor	Number of weeks made	Sample average (number scoops sold)
Vanilla	4	80
Mango	4	81
Strawberry	9	83
Coconut	9	84

(c) [2 Pts] For this part only, assume that Clara is using the UCB algorithm. Without knowing how Clara set her confidence level in the UCB algorithm, which of the following flavor(s) are possible for her to choose for the 27th week? Select all answers that apply **by filling in the square next to each correct answer**.

- ☐ Vanilla
- ■ **Mango**
- ☐ Strawberry
- ■ **Coconut**

(d) [2 Pts] Given only the information in the table above, explain why it is impossible for Clara to have used the explore-then-commit (ETC) algorithm. Your answer should be two sentences or less.

Solution: In expore-then-commit, all the arms we didn't commit to should have exactly the same number of times pulled. That isn't the case here, so we know that Clara isn't using ETC.

(e) [2 Pts] Clara talks to other ice cream vendors in Berkeley, and learns that if she had sold mango each week, she would have sold (on average) 85 scoops per week, and that any other flavor would have sold fewer scoops (on average).

What is her total regret for the 26 weeks she sold ice cream? You must include the units (e.g., seconds, inches^2, etc.) to receive full credit.

Solution: There were two ways to calculate this, both of which produce the same answer:

$$\begin{aligned}\text{Regret} &= 85 \times 26 - (80 \times 4 + 81 \times 4 + 83 \times 4 + 84 \times 9) \\ &= 5 \times 4 + 4 \times 4 + 2 \times 9 + 1 \times 9 \\ &= 63 \text{ scoops}\end{aligned}$$

6 Concentrating on Horses [6 Pts]

Charisse loves horses, so she moves to Las Vegas and bets on horse racing for several weeks. She plans to bet on hundreds of races. Let X_i represent how much money she earns betting on race i (positive values indicate winnings, and negative values indicate loss). Assume Charisse's earnings for each race are i.i.d. Let $T_n = \frac{1}{n}\sum_{i=1}^{n} X_i$ be her average earnings per race after n races.

Charisse befriends all the horse trainers and gets inside information, so her expected winnings for each race is $\$2$. To avoid having to pay out too much money, the casino will kick anyone out if their average winnings after 600 races are at least $\$10$ (in other words, if $T_{600} \geq 10$). You may assume that the casino only checks this information at exactly 600 races, and does not look at their earnings after that.

For parts (a) and (b), you must provide the tightest (i.e., smallest) correct bound to earn full credit.

Hint: the statement of Hoeffding's Inequality on the reference sheet may be helpful for some parts.

(a) [1 Pt] Given only the information above and what you learned in Data 102, provide the tightest possible upper bound on the probability that the casino kicks Charisse out for the reason above.

> **Solution:** We do not have enough information to use any concentration inequality other than Markov's inequality, which is not relevant because T_{100} can take on negative values. So, the best upper bound is 1.

(b) [2 Pts] For this part only, Charisse structures her bets so that she never **loses** more than $\$60$ on any race, and she never **wins** more than $\$180$.

Given this information and what you learned in Data 102, provide the tightest possible upper bound on the probability that the casino kicks Charisse out for the reason above. You do not need to simplify your answer to earn full credit.

Solution:

$$\begin{aligned} P(T_{100} > 10) &= P(T_{600} - 2 > 8) \\ &\leq \exp\left(\frac{-2 \times 600 \times 64}{(240)^2}\right) \end{aligned}$$

(c) [3 Pts] For this part only, Charisse structures her bets so that she never **loses** more than $\$5$ on any race, and she never **wins** more than $\$15$.

What is the smallest number of races n that she needs to bet on before she can be at least 95% confident that her average earnings will be positive? **You must express your answer as an integer for full credit**.

You may use the approximations $\log(0.05) = -3$ and/or $\log(0.95) = -0.05$.

Solution: We want $P(T_n > 0) \geq 0.95$, or equivalently $P(T_n < 0) \leq 0.05$. We can bound the probability on the LHS using Hoeffding's inequality:

$$\begin{aligned} P(T_n < 0) &= P(T_n - 2 < -2) \\ &\leq \exp\left(\frac{-2 \times n \times 4}{(15+5)^2}\right) \\ \exp\left(\frac{-2 \times n \times 4}{400}\right) &\leq 0.05 \\ n &\geq \frac{3}{8} \times 400 \\ n &\geq 150 \end{aligned}$$

This page has been intentionally left blank. No work on this page will be graded.

7 Congratulations [0 Pts]

Congratulations! You have completed Midterm 2.

- **Make sure that you have written your student ID number on *every other page* of the exam.** You may lose points on pages where you have not done so.
- Also ensure that you have **signed the Honor Code** on the cover page of the exam for 1 point.
- If more than 10 minutes remain in the exam period, you may hand in your paper and leave. If $\leq$ 10 minutes remain, please **sit quietly** until the exam concludes.

[Optional, 0 pts] Draw a picture or cartoon that's related to your favorite thing you've learned in Data 102 so far.

Midterm 2 Reference Sheet

Useful Distributions:

Distribution	Support	PDF/PMF	Mean	Variance	Mode
$X \sim \text{Poisson}(\lambda)$	$x = 0, 1, 2, \ldots$	$\frac{\lambda^x e^{-\lambda}}{x!}$	λ	λ	$\lfloor \lambda \rfloor$
$X \sim \text{Binomial}(n, p)$	$x \in \{0, 1, \ldots, n\}$	$\binom{n}{x} p^x (1-p)^{1-x}$	np	$np(1-p)$	$\lfloor (n+1)p \rfloor$
$X \sim \text{Beta}(\alpha, \beta)$	$0 \leq x \leq 1$	$\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha}{\alpha+\beta} \frac{\beta}{\alpha+\beta} \frac{1}{\alpha+\beta+1}$	$\frac{\alpha-1}{\alpha+\beta-2}$
$X \sim \text{Gamma}(\alpha, \beta)$	$x \geq 0$	$\frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$	$\frac{\alpha}{\beta}$	$\frac{\alpha}{\beta^2}$	$\frac{\alpha-1}{\beta}$
$X \sim \mathcal{N}(\mu, \sigma^2)$	$x \in \mathbb{R}$	$\frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)$	μ	σ^2	μ
$X \sim \text{Exponential}(\lambda)$	$x \geq 0$	$\lambda \exp(-\lambda x)$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	0
$X \sim \text{InverseGamma}(\alpha, \beta)$	$x \geq 0$	$\frac{\beta^\alpha}{\Gamma(\alpha)} x^{-\alpha-1} e^{-\beta/x}$	$\frac{\beta}{\alpha-1}$	$\frac{\beta^2}{(\alpha-1)^2(\alpha-2)}$	$\frac{\beta}{\alpha+1}$

Conjugate Priors: For observations x_i, $i = 1, \ldots, n$:

Likelihood	Prior	Posterior
$x_i \mid \theta \sim \text{Bernoulli}(\theta)$	$\theta \sim \text{Beta}(\alpha, \beta)$	$\theta \mid x_{1:n} \sim \text{Beta}\left(\alpha + \sum_i x_i, \beta + \sum_i (1 - x_i)\right)$
$x_i \mid \mu \sim \mathcal{N}(\mu, \sigma^2)$	$\mu \sim \mathcal{N}(\mu_0, 1)$	$\mu \mid x_{1:n} \sim \mathcal{N}\left(\frac{\sigma^2}{\sigma^2+n}\left(\mu_0 + \frac{1}{\sigma^2}\sum_i x_i\right), \frac{\sigma^2}{\sigma^2+n}\right)$
$x_i \mid \lambda \sim \text{Exponential}(\lambda)$	$\lambda \sim \text{Gamma}(\alpha, \beta)$	$\lambda \mid x_{1:n} \sim \text{Gamma}\left(\alpha + n, \beta + \sum_i x_i\right)$
$x_i \mid \lambda \sim \text{Poisson}(\lambda)$	$\lambda \sim \text{Gamma}(\alpha, \beta)$	$\lambda \mid x_{1:n} \sim \text{Gamma}\left(\alpha + \sum_i x_i, \beta + n\right)$
$x_i \mid \lambda \sim \mathcal{N}(\mu, \sigma^2)$	$\sigma \sim \text{InverseGamma}(\alpha, \beta)$	$\sigma \mid x_{1:n} \sim \text{InverseGamma}\left(\alpha + n/2, \beta + \left(\sum_{i=1}^n (x_i - \mu)^2\right)/2\right)$

Generalized Linear Models

Regression	Inverse link function	Likelihood
Linear	identity	Gaussian
Logistic	sigmoid	Bernoulli
Poisson	exponential	Poisson
Negative binomial	exponential	Negative binomial

Hoeffding's Inequality: If $X_1, \ldots, X_n$ are independent random variables bounded between a and b, then

$$P\left(\frac{1}{n}\sum_{i=1}^n (X_i - E[X_i]) > t\right) \leq \exp\left(-\frac{2nt^2}{(a-b)^2}\right)$$

$$P\left(\frac{1}{n}\sum_{i=1}^n (X_i - E[X_i]) < -t\right) \leq \exp\left(-\frac{2nt^2}{(a-b)^2}\right)$$

$$P\left(\left|\frac{1}{n}\sum_{i=1}^n (X_i - E[X_i])\right| > t\right) \leq 2\exp\left(-\frac{2nt^2}{(a-b)^2}\right)$$