DS 102 Discussion 7

Wednesday, March 16, 2022

1. **Potential Outcomes**

   A fundamental framework of causal inference is *potential outcomes*– the outcomes for a study unit given the potential treatments. We will use the notation $Y_i(1)$ to denote the i-th unit's outcome if they were given the treatment, and $Y_i(0)$ to denote the i-th unit's outcome if they were given the control. There are two assumptions that are necessary to proceed under the potential outcomes framework:

   1. *No Interference*: Unit i's potential outcomes don't depend on other unit's treatments.
   2. *Consistency*: The treatment is well-defined in the study (i.e. there is only one version of the treatment).

   These two assumptions are commonly grouped together, jointly called the *Stable Unit Treatment Value Assumption* (SUTVA).

   (a) *Scenario 1: A Marketing Campaign*

   Suppose you are running a study to determine the causal effect of a marketing campaign on social media on product sales. Assuming that other aspects of the experiment design are correct (e.g. randomized treatment assignment), which assumption(s) of SUTVA may likely be violated in this setting?

   (b) *Scenario 2: A Public Health Decision*

   Suppose you are running a study to determine the causal effect of restricting public transportation on COVID-19 transmission. Which assumption(s) of SUTVA may likely be violated in this setting?

An $n \times 2$ table of potential outcomes, sometimes called the *Science Table*, summarizes the potential outcomes of every subject in a study:

i	$Y_i(1)$	$Y_i(0)$
1	$Y_1(1)$	$Y_1(0)$
2	$Y_2(1)$	$Y_2(0)$
$\vdots$	$\vdots$	$\vdots$
n	$Y_n(1)$	$Y_n(0)$

Figure 1: The Science Table for n subjects

(c) *The Fundamental Problem of Causal Inference*

After we perform a study, how many entries are known in the Science Table? Why can we think of causal inference as a missing data problem?

(d) *Observed Outcomes vs. Potential Outcomes*

Let Y_i be the observed outcome of subject i, defined as:

$$Y_i = \begin{cases} Y_i(1), \text{ if } Z_i = 1 \\ Y_i(0), \text{ if } Z_i = 0 \end{cases}$$

where Z_i is the treatment indicator. Show that $Y_i = Y_i(0) + \tau_i Z_i$.

Hint: The individual treatment effect is defined as $\tau_i = Y_i(1) - Y_i(0)$.

2. **Causal DAGs**

Earlier in Data 102, we learned about graphical models, which are diagrams that express the relationships between a set of random variables. Now, we'll extend this idea to causal inference in the form of *Causal DAGs*: graphical models in the form of directed acyclic graphs where arrows indicate *causal relationships* between variables.

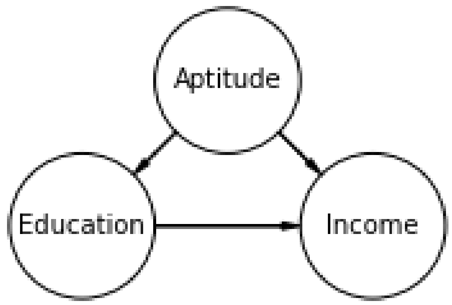


Figure 2: A Causal DAG showing a confounding variable, *Aptitude*

(a) *Drawing a Causal DAG*

Consider the following variables:

- **L**: Location of garden
- **S**: Soil Quality
- **Z**: Rainfall (High or Low)
- **Y**: Number of flowers grown
- **P**: Amount of Pollen on flowers
- **I**: Number of Insects on flowers

For the variables defined in the problem, draw the Causal DAG which best captures their causal relationships.

(b) *Estimating Treatment Effects with a Collider*

Let's say we are interested in estimating the causal effect of rainfall Z on the number of flowers grown Y, without knowing the true causal relationships. You collect data on Z, Y and covariate I, presented in the following plots:

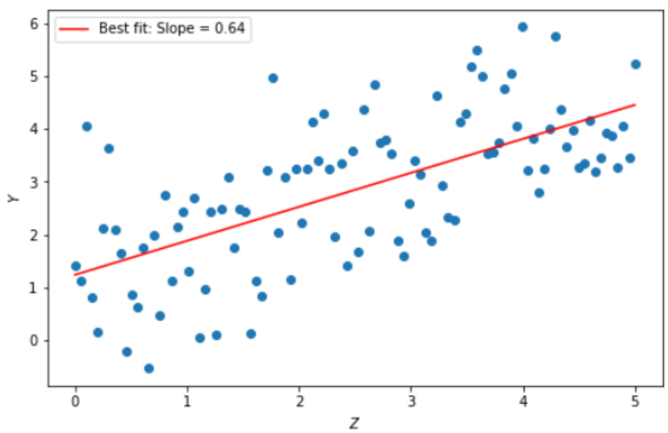


Figure 3: $Y \sim Z$

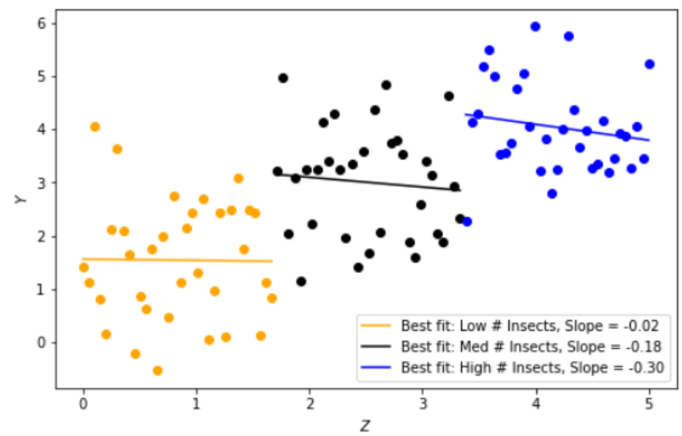
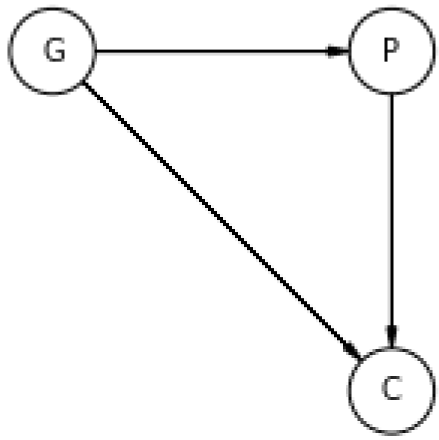


Figure 4: $Y \sim Z$ controlling for I

Use these plots to estimate the treatment effect of Z on Y, with and without controlling for I. Which estimate is more accurate? Assume there are equivalent numbers of units within each strata of I.

(c) *Estimating Causal Effects based on a DAG*

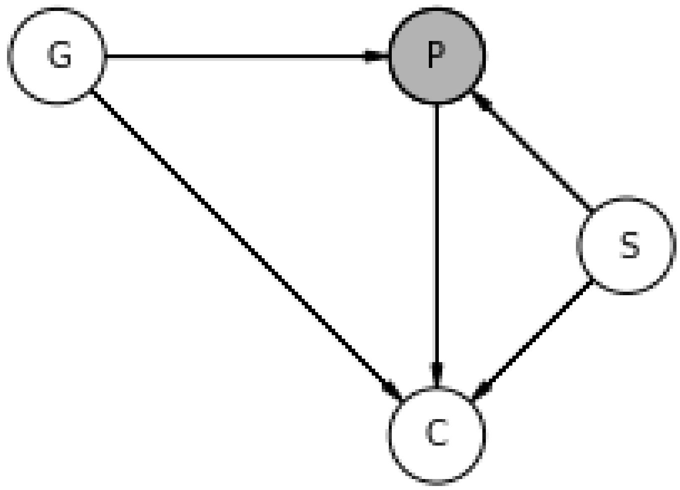
Now, we have a new set of variables $\{G, P, C\}$, where G refers to grandparent education level, P refers to parent education level, and C refers to the child education level. Suppose the causal DAG for these variables looks like this:



You are tasked with estimating the direct causal effect of G on C. Explain how you could achieve this.

(d) *Collider Bias and Adjustment Sets*

Based on your answer to the previous part, you choose to collect data on $\{G, P, C\}$ which was stratified on P. However, now suppose the true causal DAG looks like this,



where S is an unknown confounder which represents the school district that the family lives in. Explain why this is a problem for estimating the direct effect of G on C. Can you think of ways to eliminate this bias by collecting additional data?