LECTURE NINE

① More PyMC examples:

→ Normal Mean Estimation

→ Outliers

→ Gaussian Mixture Model

② Rejection Sampling

"Markov Chain Monte Carlo."

Measurement Problem

θ : unknown physical quantity

Goal: Measure θ

$n = 15$

17.62 17.62 17.615 17.62
17.61 17.62 17.625 17.62
17.61 17.615 17.61 17.605

17.61
17.62
17.61

Standard Method:

Take the mean of the measurements

$\bar{y}$: point estimate of θ.

standard deviation of data points:

$$\underbrace{\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2}}_{\hat{\sigma}}$$

Uncertainty in the point estimate

$$= \frac{\hat{\sigma}}{\sqrt{n}}$$

$$\bar{y} \pm \frac{\hat{\sigma}}{\sqrt{n}}$$

Confidence Interval for θ:

$$\left[\bar{y} - 2\frac{\hat{\sigma}}{\sqrt{n}}, \bar{y} + 2\frac{\hat{\sigma}}{\sqrt{n}}\right]$$

θ : parameter

Data : $y_1 \dots y_n$

Prior : $\theta \sim \text{Unif}(0, 30)$,

Likelihood : $y_1 \dots y_n \overset{iid}{\sim} N(\theta, \sigma^2)$

$\log \sigma \sim \text{Unif}(-10, 10)$

$\text{Unif}(-10, 0)$

$\boxed{\theta, \sigma}$

Dealing with Outliers

Data : $y_1 \dots y_n$

$z_1 \dots z_n$ binary

θ

Model : $z_1 \dots z_n \overset{iid}{\sim} \text{Ber}(w)$

$$y_i \mid \begin{matrix} z_i = 0 \\ \theta, \sigma \end{matrix} \sim N(\theta, \sigma^2)$$

$$y_i \mid \begin{matrix} z_i = 1 \\ \theta, \sigma \end{matrix} \sim N(0, 100^2)$$

$\theta \sim \text{Unif}[0, 30]$, $\log \sigma \sim \text{Unif}(-10, 0)$

$w \sim \mathrm{Unif}[0,1]$

REJECTION SAMPLING

$$f_{Target}(u)$$

$$f_{Proposal}(u)$$

Assumption: $\dfrac{f_{Target}(u)}{f_{Proposal}(u)} \leq M$

for some known M.

Method: ① Generate samples from $f_{Proposal}$

② Accept or Reject them with some $\underbrace{\text{probability}}$

$$\frac{f_{Target}(u)}{M \cdot f_{Proposal}(u)}$$