# Midterm 2 Reference Sheet

**Useful Distributions:**

| Distribution | Support | PDF/PMF | Mean | Variance | Mode |
|---|---|---|---|---|---|
| $X \sim \text{Poisson}(\lambda)$ | $x = 0, 1, 2, \ldots$ | $\frac{\lambda^x e^{-\lambda}}{x!}$ | $\lambda$ | $\lambda$ | $\lfloor \lambda \rfloor$ |
| $X \sim \text{Binomial}(n, p)$ | $x \in \{0, 1, \ldots, n\}$ | $\binom{n}{x} p^x (1-p)^{1-x}$ | $np$ | $np(1-p)$ | $\lfloor (n+1)p \rfloor$ |
| $X \sim \text{Beta}(\alpha, \beta)$ | $0 \leq x \leq 1$ | $\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}$ | $\frac{\alpha}{\alpha+\beta}$ | $\frac{\alpha}{\alpha+\beta}\frac{\beta}{\alpha+\beta}\frac{1}{\alpha+\beta+1}$ | $\frac{\alpha-1}{\alpha+\beta-2}$ |
| $X \sim \text{Gamma}(\alpha, \beta)$ | $x \geq 0$ | $\frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$ | $\frac{\alpha}{\beta}$ | $\frac{\alpha}{\beta^2}$ | $\frac{\alpha-1}{\beta}$ |
| $X \sim \mathcal{N}(\mu, \sigma^2)$ | $x \in \mathbb{R}$ | $\frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)$ | $\mu$ | $\sigma^2$ | $\mu$ |
| $X \sim \text{Exponential}(\lambda)$ | $x \geq 0$ | $\lambda \exp(-\lambda x)$ | $\frac{1}{\lambda}$ | $\frac{1}{\lambda^2}$ | $0$ |

**Conjugate Priors:** For observations $x_i$, $i = 1, \ldots, n$:

| Likelihood | Prior | Posterior |
|---|---|---|
| $x_i \vert \theta \sim \text{Bernoulli}(\theta)$ | $\theta \sim \text{Beta}(\alpha, \beta)$ | $\theta \vert x_{1:n} \sim \text{Beta}\left(\alpha + \sum_i x_i, \beta + \sum_i (1 - x_i)\right)$ |
| $x_i \vert \mu \sim \mathcal{N}(\mu, \sigma^2)$ | $\mu \sim \mathcal{N}(\mu_0, 1)$ | $\mu \vert x_{1:n} \sim \mathcal{N}\left(\frac{\sigma^2}{\sigma^2+n}\left(\mu_0 + \frac{1}{\sigma^2}\sum_i x_i\right), \frac{\sigma^2}{\sigma^2+n}\right)$ |
| $x_i \vert \lambda \sim \text{Exponential}(\lambda)$ | $\lambda \sim \text{Gamma}(\alpha, \beta)$ | $\lambda \vert x_{1:n} \sim \text{Gamma}\left(\alpha + n, \beta + \sum_i x_i\right)$ |

## Generalized Linear Models

| Regression | Inverse link function | Likelihood |
|---|---|---|
| Linear | identity | Gaussian |
| Logistic | sigmoid | Bernoulli |
| Poisson | exponential | Poisson |
| Negative binomial | exponential | Negative binomial |

Some powers of $e$:

| $x$ | 0.05 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $y = e^x$ | 1.05 | 1.11 | 1.22 | 1.35 | 1.49 | 1.65 | 1.82 | 2.01 | 2.23 | 2.46 | 2.72 |