Data 102 (Fall 2022) **Names:**
Discussion Worksheet 10
11/04/2022

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1. Frame each scenario as a multi-armed bandits problem. Describe in words what the arms and the payoffs are.

   (a) You're trying to figure out which restaurant on Euclid is your favorite, so every day for a month you get lunch at one of the restaurants on Euclid.

   Solution: Arms: The restaurants. Payoff: How much you like each restaurant.

   (b) You're trying to figure out what time of day your shower is most likely to have warm water, so you take a shower at a different time of day for a sequence of days and observe the temperature of the water.

   Solution: Arms: Times of day. Payoff: 1 if the water is warm, 0 if not.

2. (a) Let $X_1, ..., X_n$ be i.i.d. random variables with variance σ^2, mean μ, and let $Z = \frac{1}{n}\sum_{i=1}^{n} X_i$. What is Var$(Z)$?

   Solution:

$$\text{Var}(Z) = \frac{\sigma^2}{n}$$

   (b) Using Chebyshev's inequality, find an upper bound on the probability that Z is at least ϵ-far away from μ, i.e. on $\mathbb{P}[|Z - \mu| \geq \epsilon]$.

   Solution:

$$\mathbb{P}(|Z-\mu| \geq \epsilon) = \mathbb{P}\left(|Z-\mu| \geq (\sigma/\sqrt{n})\frac{\epsilon}{\sigma/\sqrt{n}}\right) \leq \frac{\sigma^2}{n\epsilon^2}.$$

3. Suppose you're trying to find the fastest driving route from Berkeley to San Francisco at 8 am out of 3 different routes. To try to determine this, every day for 5 days you drive from Berkeley to San Francisco at 8 am, choosing one of the routes each day.

You model the route you choose on day t as a random variable A_t, which can take values in the set $\{1, 2, 3\}$. You model the time it takes to drive the route chosen on day t as a random variable $D_t \sim \mathcal{P}_{A_t}$.

(a) What are the arms?

Solution: Routes $\{1, 2, 3\}$.

(b) The expected time it takes to travel routes 1, 2, and 3 respectively is 30, 45, and 60 min respectively. The time it takes you to travel between Berkeley and SF on the five days is 75, 40, 50, 40, and 70 min respectively. What is your regret?

Solution:

$$R = \left(\sum_{t=1}^{5} D_t\right) - 5(\min_i \mathbb{E}[D_{a_i}]) = (75+40+50+40+70) - 5(30) = 275 - 150 = 125\text{min.}$$

(c) On those five days, you took routes 3,2,2,1,3 respectively. What is the pseudo-regret?

Solution:

$$pR = \mathbb{E}_{\mathcal{P}_{A_t}}\left[\sum_{t=1}^{5} D_t\right] - 30(\min_i \mathbb{E}[D_{a_i}]) = (60+45+45+30+60) - 5(30) = 240 - 150 = 90\text{min.}$$

(d) Is the regret a random quantity? How about pseudo-regret?

Solution: Both are random. In the regret, D_t are random, and in the pseudo-regret, $\mathbb{E}[D_t]$ are still random in A_t.

4. Let X be the sum of 20 i.i.d. Poisson random variables $X_1, \ldots, X_{20}$ with $\mathbb{E}[X_1] = 1$. Use the following techniques to upper bound $\Pr(X \geq 26)$.

   (a) Markov's Inequality

   (b) Chebyshev's Inequality

   (c) Chernoff Bound

   Hint 1: The MGF of a mean-λ Poisson random variable X is

$$M_X(t) = e^{\lambda(e^t - 1)}.$$

   Hint 2: if X, Y are independent Poisson random variables with parameter λ_1, λ_2, then $Z = X + Y$ is a Poisson random variable with parameter $\lambda_1 + \lambda_2$.

   Solution:

   (a) Using Markov's Inequality:

$$\Pr(X \geq 26) \leq \frac{20}{26} \approx 0.769.$$

   (b) Using Chebyshev's Inequality:

$$\Pr(X \geq 26) \leq \Pr(|X - 20| \geq 6) \leq \frac{20}{36} \approx 0.5556.$$

   (c) Using the Chernoff Bound we have

$$\Pr(X \geq 26) \leq \min_{s \geq 0} e^{-26s} e^{20(e^s - 1)} = \min_{s \geq 0} e^{-26s + 20e^s} e^{-20}.$$

Feedback Form

On a scale of 1-5, where 1 = much too slow and 5 = much too fast, how was the pace of the discussion section?

1 2 3 4 5

Which problem(s) did you find most useful?

Which were least useful?

Any other feedback?