

1. (Data 140 Exercise 15.6.1) Let  $X$  have density given by

$$f(x) = \begin{cases} c(1 - x^2), & -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find:

(a)  $c$

(b) the cdf of  $X$

(c)  $P(|X| > 0.5)$

2. Suppose  $X$  and  $Y$  are independent random variables. Which of the following are necessarily true?

(a)  $\mathbb{E}[XY] = \mathbb{E}[X]\mathbb{E}[Y]$

(b)  $\mathbb{E}[e^{X+Y}] = \mathbb{E}[e^X]\mathbb{E}[e^Y]$

(c)  $\text{Var}[X + Y] = \text{Var}[X] + \text{Var}[Y]$

(d)  $\text{Var}[XY] = \text{Var}[X] \text{Var}[Y]$

3. Consider the following linear regression model:

$$\hat{y}_i = \theta_0 + \theta_1 x_{i,1} + \theta_2 x_{i,2}$$

Suppose that we observe the data:

$$y_1 = 1, \quad x_1 = (2, 1)$$

$$y_2 = 2, \quad x_2 = (2, -1)$$

$$y_3 = 3, \quad x_3 = (0, -1)$$

$$y_4 = 4, \quad x_4 = (0, 1)$$

(a) What is the least-squares estimate for  $\theta$ ?

(b) What is the predicted value  $\hat{y}$  when  $x = (1, 1)$ ?

(c) What is the RMSE (root mean-squared error)?