DS 102 Discussion 10

Wednesday, November 11th, 2020

Examples on Nash Equilibrium

In class, we discussed the Nash equilibrium in two-player games. Denote the action space of player $i (i \in \{1, 2\})$ as $\mathcal{A}_i$. The payoff function (outcome) for player i is a function that maps the vector of actions taken by player $1, 2$ to some real value $u_i : \mathcal{A}_1 \times \mathcal{A}_2 \mapsto \mathbb{R}$. Each player i would like to maximize their own payoff function $u_i(a_1, a_2)$. We say the action pair (a_1^*, a_2^*) for the two players is a Nash equilibrium if $\forall a_1' \in \mathcal{A}_1, u_1(a_1^*, a_2^*) \geq u_1(a_1', a_2^*)$ and $\forall a_2' \in \mathcal{A}_2, u_2(a_1^*, a_2^*) \geq u_2(a_1^*, a_2')$.

The definition of Nash Equilibrium can be extended to multi-player setting. Assume we have n players in total. Denote the payoff function for player i as $u_i : \mathcal{A}_1 \times \mathcal{A}_2 \times \cdot \times \mathcal{A}_n \mapsto \mathbb{R}$. Denote $a_{-i} = (a_1, \cdots, a_{i-1}, a_{i+1}, \cdots, a_n)$ as the vector of actions of all players except for player i. We say the action pair $(a_1^*, a_2^*, \cdots, a_n^*)$ is a Nash equilibrium if for all $i \in [n], a_i' \in \mathcal{A}_i$, we have $u_i(a_i^*, a_{-i}^*) \geq u_i(a_i', a_{-i}^*)$.

Consider the following games and provide one Nash equilibrium for each of them.

(a) A two-player non-zero-sum game with payoff matrix as below.

		Player 1: 0	Player 1: 1
Player 2	0	$(3, 3)$	$(1, 0)$
	1	$(2, 2)$	$(5, 5)$

(b) A two-player zero-sum game. The action space for both players is $\mathcal{A}_i = \mathbb{R}$. We use $X, Y \in \mathbb{R}$ to denote the action of player 1 and 2, separately. The payoff function for player 1 is $u_1(X, Y) = Y^2 - X^2 + 2XY + 2X$. The payoff function for player 2 is $u_2(X, Y) = -u_1(X, Y) = -Y^2 + X^2 - 2XY - 2X$.

(c) (Optional) A n-player single-item second-price auction. Denote the private valuation of the i-th bidder as $v_i \in \mathbb{R}^+$, the bid of the i-th bidder as $b_i \in \mathbb{R}^+$. The payoff function for bidder i is $u_i(b_1, b_2 \cdots, b_n) = (v_i - \max_{j \neq i} b_j) \cdot 1(b_i \geq \max_{j \neq i} b_j)$. (Take some time to convince yourself that this payoff function is exactly the gain of bidder i from second-price auction.)